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ASTRONOMY AND ASTROPHYSICS

The puzzling theoretical predictions for the luminosity of clumping He burning stars

V. Castellani[1,2], S. Degl'Innocenti[1,2], L. Girardi[3], M. Marconi[4], P.G. Prada Moroni[1], A. Weiss[5]

[1] Dipartimento di Fisica, Universitá di Pisa, piazza Torricelli 2, I-56126 Pisa, Italy
[2] Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, via Livornese 1291, I-56010 S. Piero a Grado, Pisa, Italy
[3] Dipartimento di Astronomia, Vicolo dell'Osservatorio 5, I-35122 Padova, Italy
[4] Osservatorio Astronomico di Capodimonte, via Moiariello 16, I-80131 Napoli, Italy
[5] Max Planck Institut für Astrophysics, K. Schwarzschild Strasse 1, D-85740 Garching, München, Germany

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Abstract. This paper deals with theoretical predictions for He burning models in a range of masses covering the so-called Red Giant Branch phase transition. Taking as a guideline the observational constraints given by Hipparcos parallaxes to the predicted luminosity of models originated from Red Giant progenitors with He core undergoing electron degeneracy, we compare models by various authors as recently appeared in the literature, disclosing sensitive differences in the predicted luminosity. The "solidity" of these theoretical predictions is investigated by exploring the effects of varying the assumptions about the efficiency of core overshooting or the amount of mass loss, giving quantitative estimates of the related uncertainties. However, one finds that theoretical predictions concerning the luminosity of the red giant clump in the Hipparcos sample is scarcely affected by these mechanisms. **A comparison among theoretical predictions as recently given by different authors convincingly demonstrates that the different luminosity predictions are the natural results of evolutionary codes with different - but all reasonable- input physics.** In this context observations suggests that stellar models based on the "most updated" input physics are possibly overestimating the luminosity of these structures, raising doubts on several current predictions concerning the luminosity of HB stars in galactic globulars.

Key words: Stars:evolution, Stars: Hertzsprung-Russell diagram

1. Introduction

Hipparcos parallaxes for clumping He burning Red Giants in the solar neighborhood have recently raised a large interest in the astronomical community, providing a new valuable (though controversial) tool to approach the problem of Magellanic Clouds distances (see, e.g., Udalski et al. 1998, Stanek, Zaristski et al. 1998, Cole 1998, Girardi et al. 1998). However, on theoretical grounds one has to notice that these parallaxes provide us for the first time with direct observational evidences for the luminosity of He burning stars whose Red Giant progenitors experienced electron degeneracy in the stellar core. Therefore providing a relevant test for the rather sophisticated input physics supporting current theoretical predictions concerning galactic globulars and, more in particular, concerning some relevant issues as the luminosity of RR Lyrae stars and the ages of halo stellar clusters.

Such a test appears now of particular interest because of the growing rumor about a possible overluminosity of theoretical models for He burning stars with degenerated progenitors. As a matter of example, Pols et al. (1998) found that the He clump luminosity in the open cluster M67 appears 0.2 - 0.3 mag fainter than predicted on the basis of their evolutionary computations. Such a discrepancy appears larger than the expected uncertainties on the bolometric correction and it appears further supported by independent evolutionary computations recently presented by Castellani et al. (1999), hereinafter C99, which have discussed the severe difficulties in fitting the CM diagram of that clusters. However, in the meantime, Girardi et al. (1998) presented a careful discussion on the luminosities of Hipparcos He burning giants, which were found in splendid agreement with the adopted theoretical predictions.

To throw light on such a scenario, in the next section we will discuss current predictions about the luminosity of He burning stars in a suitable mass range, disclosing the occurrence of sensitive differences among various authors. Section 3 will be devoted to a preliminary investigation of the uncertainties in theoretical results due to uncertainties on both the amount of extramixing from convective cores and the amount of mass loss in the pre-He burning phases. On this basis, in section 4 we will compare theo-

Send offprint requests to: S. Degl'Innocenti, scilla@astr18pi.difi.unipi.it

retical results with the observed mean luminosity of the red giant clump in the Hipparcos sample, concluding for the possible need of a revision of the "most updated" input physics used in recent evolutionary models. The origin of differences in the predicted luminosities are finally discussed in section 5, by comparing the results of selected evolutionary codes to the light of the adopted physical ingredients. A short section of concluding remarks will close the paper.

2. Comparing theories

According to Girardi et al. (1998), the Hipparcos sample of neighboring He burning giants is largely populated by stars with masses below or in the range of the so-called Red Giant Branch phase transition (RGB-pt). Since the pioneering paper by Sweigart et al. (1990) this range of stellar masses has been the subject of several careful evolutionary investigations. As well known, as we go from stars with M$\sim$1 $M_\odot$ to higher masses, we progressively find H-shell burning stars with a lower degree of electron degeneracy in their cores. Eventually, He is quietly ignited in the center of the structure. As a consequence, the mass of the He core at the He ignition progressively decreases, reaching a minimum at a star mass which depends on the original chemical composition. After this minimum, the He core grows again with mass, following the increasing size of the central convective core in the Main Sequence structures.

Fig. 1 gives selected quantities concerning the behavior of He burning models across the RGB-pt as computed for Z=0.02 and the two alternative assumptions Y=0.27 or 0.23. All models have been computed according to the theoretical scenario already presented in C99, which incorporates all the most recent evaluations of the input physics. As everywhere in the following, quantities given in Fig.1 refer to the first model which, after igniting central He, has already reached the HR diagram location where it will spend the major phase of central He burning. Let us here notice that the luminosity of these He burning models is not related only to the mass of the He core. Initially this luminosity increases in spite of the decreasing He core, since the increased efficiency of the H burning shell overcomes the decreased output of He-burning reactions. However, eventually the decrease of the He core dominates and the luminosity reaches its minimum, whereas the lifetime in the central helium burning phase increases following the decrease of the efficiency of the He burning reactions. The subject of the RGB-pt has been already widely debated in the literature (see, e.g., Renzini & Buzzoni 1986, Corsi et al. 1994, Girardi & Bertelli 1998 and references therein) and here it does not deserve further comments.

However, Fig. 2 compares the luminosities given in Fig.1 with similar results but by Girardi et al. (1998: G98 hereinafter). To our surprise, one finds that G98 luminosities appear systematically fainter than in C99 by about

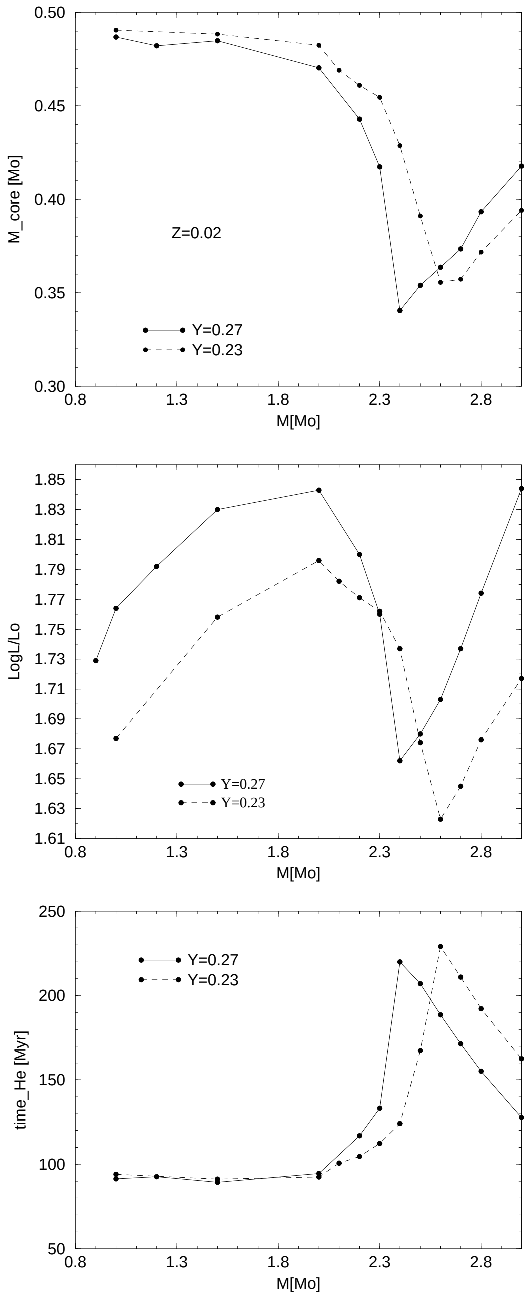


Fig. 1. Selected quantities depicting the behavior of stellar models across the Red Giant Transition as computed for solar metallicity (Z=0.02) and for the two choices about the amount of original helium Y=0.27 or Y=0.23. Top to bottom: i) the mass of the He core at the beginning of the major phase of central He burning, ii)the predicted luminosity of the same models and, iii) the predicted He burning lifetime.

$\Delta\log L/L_\odot \sim 0.1 \div 0.15$, which is far from being a negligible amount and, in turn, it appears of the right amount to solve the already quoted M67 discrepancy.

Such an evidence prompted us to investigate similar data in the literature, aiming to find the origin of such

a difference. To this purpose, the same Fig. 2 shows the predictions already given in the literature on the basis of of the Frascati or Padua evolutionary codes before the last updating of the input physics. The figure gives the comforting evidence that luminosities from Castellani et al. (1992) appear in rather good agreement with similar data by Bressan et al. (1993). As we will further discuss in the next section, the slight underluminosity and the little difference in the RGB-pt mass of Bressan et al. models is only the expected consequence of their adoption of a moderate core overshooting scenario.

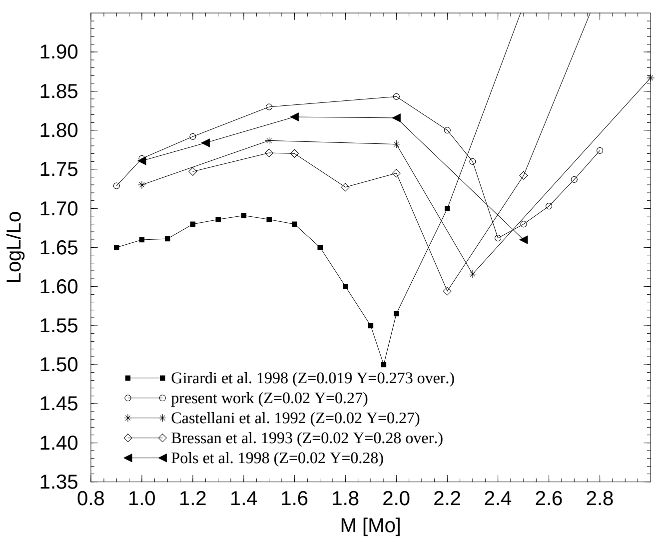


Fig. 2. He clump luminosity as a function of the stellar mass for Z=0.02 and Y≈0.27 by various authors, as labeled; "over" marks evolutions with core overshooting.

The same figure shows that the updated input physics adopted in C99 has the effect of increasing the luminosity of the models with a degenerate progenitors, according to the discussion already given in Cassisi et al. (1998) and in reasonable agreement with stellar models recently presented by Pols et al. (1998). However, one also finds that the new input physics in the Padua models (Girardi & Bertelli 1998, Girardi et al. 1999) has the opposite effect, sensitively decreasing the predicted luminosities. As a whole, one finds that uncertainties on predicted luminosities can be even larger than ΔlogL$\sim$0.1, leaving an unpalatable uncertainty in the current evolutionary scenario.

On general grounds, one expects that the quoted differences are the results of differences either in the input physics or in the assumptions about the efficiency of macroscopic mechanisms, like core overshooting, which can affect the evolution of stellar structures. To discuss this point, in the next sections we will investigate the range of variability in current theoretical predictions, as produced by the various assumptions governing the evolutionary behaviour.

3. Theoretical uncertainties in predicted luminosity.

Making reference to the set of models presented in C99, in this section we will explore the influence on central He burning models of several assumptions concerning these structures, namely, i) the efficiency of core overshooting mechanisms, and, ii) the effect of mass loss. In this way we aim to reach a clear insight on the "solidity" of the result one is dealing with in the literature.

Fig. 3 (upper panel) shows the effect on the model luminosity of selected choices about the efficiency of core overshooting when the original stellar mass is varied between 1 and 3 $M_\odot$, while the lower panel in the same figure adopts the G98 representation to show the run of the same models in the HR diagram. Labels in these figure give the adopted amount of extramixing (in unity of the local pressure scale height) around the convective cores. In passing, note that comparison between this figure and Fig.1 gives the already known evidence that the RGB phase transition shifts to lower masses as the metallicity decreases.

As already known, one finds that overshooting decreases the mass of the RGB-pt (although it then occurs at a larger age) and, correspondingly, that the maximum luminosity reached by the models before the transition decreases. However, one finds that for moderate amounts of overshooting such a decrease is rather small and, in any case, models with masses of the order of 1.2 $M_\odot$ or lower are little affected by such a mechanism. In addition the minimum luminosity attained at the RGB-pt varies by only ΔlogL/$L_\odot \approx 0.03$ between a standard model and a model with l_{ov}=0.25.

Thus the differences in the assumptions about the efficiency of overshooting can hardly be at the origin of the differences in Fig. 2 and, in turn, they cannot be used to reconcile Pols et al. (1998) or C99 computations with M67 or Hipparcos constraints.

The effect of mass loss deserves a bit more discussion. Here we will assume that mass loss occurs in the advanced phase of H shell burning, so that the internal structure of the He burning star is not affected by such an occurrence, which only decreases the amount of envelope surrounding the central He core. Under this assumption, the effect of mass loss on He burning models can be easily computed by simply decreasing the envelope of the constant-mass model. Fig. 4 maps the effect in the HR diagram of different amount of mass loss from the selected models. The behavior depicted by data in this figure can be easily understood as follows:

i) As long as models develop strong electron degeneracy (i.e., for masses lower or of the order of 1.5 $M_\odot$) the mass of the He core at the He ignition is the result of RGB evolution. As a consequence it is largely independent of the evolving mass and, in addition, it is little affected by mass loss (see, e.g., the discussion in Castellani & Castellani 1993), thus He burning models with mass losing progen-

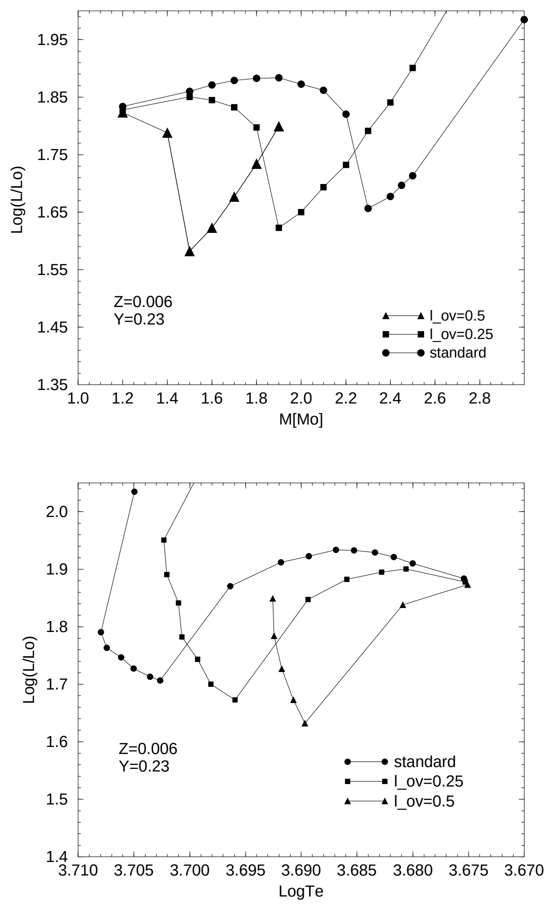


Fig. 3. Upper panel: The luminosity of He burning models with Z=0.006 and Y=0.23 as a function of the stellar mass and for the various labeled assumptions about the efficiency of core overshooting (l_ov), see text. Lower panel: the run of the same models but in the HR diagram.

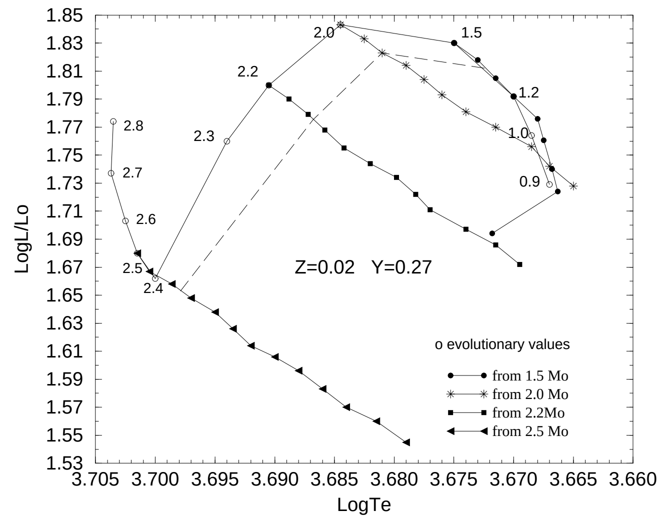


Fig. 4. The HR diagram location of He burning structures under different assumptions about the amount of mass loss in the pre He burning phase. Open circles give the location of models without mass loss with the labeled values of the stellar masses. The shift in the HR diagram expected by decreasing the total mass by step of 0.1 $M_\odot$ is shown for the four labeled values of the original mass. Dashed line gives the expected distribution for models having lost 10% of their original mass.

itors behave like models without mass loss but starting their evolution with the actual mass of the He burning model. In this case the evolutionary behavior with mass loss can be easily predicted in terms of canonical models without mass loss.

ii) For larger stellar masses the He core at the He ignition is largely connected to the extension of the convective cores in the previous MS structures. As a consequence, the mass of the He core in He burning structures is now a sensitive function of the original stellar mass, whereas it keeps being little affected by mass loss, which mainly occurs in the post MS phases. In such case theoretical expectations for the luminosity abandon the location of canonical models, as shown in Fig. 4.

As an example, Fig. 4 gives theoretical predictions assuming for all stars a common mass loss of 10%.

This is intended to be a useful illustration of the effect we get assuming reasonable mass loss along the RGB. However, one should keep in mind that the real situation may well be more complicate. Assuming the Reimers (1975) mass-loss rates, for instance, one finds that stars of higher mass lose less mass along the RGB. This is so because they have lower RGB-tip luminosities, evolve at higher effective temperatures, and have a shorter RGB lifetime. In this case, stars more massive than about 1.5 $M_\odot$ would lose negligible amounts of mass on the RGB, contrarily to the lowest-mass ones (see fig. 6 in Girardi 1999). In the context of the present discussion, this means that mass-loss is expected to be less effective exactly in the mass range in which it would more affect the luminosity of the He-burning models, i.e. in the left part of Fig. 4.

4. The Hipparcos constraints

In Sect. 3 we have explored the influence of physical effects which are badly understood and the treatment of which is usually done in parametrized form in stellar evolution codes. Having discussed the scenario of these theoretical uncertainties, let us here attempt to use Hipparcos constraints to test the already quoted different predictions concerning the luminosities of He burning stars. Following the careful discussion given by G98 about the clump population, one finds that for any reasonable assumption about the age spread of field stars one predicts the bulk of the clump to be populated by stars below the transition mass, whose luminosity is practically insensitive to assumptions about the efficiency of overshooting. Coming back to Fig.2, one finds that at a solar metallicity C99 predictions give for these stars a luminosity larger than G98 by about ΔlogL=0.1. More in general, comparison

of data in G98 and C99 shows that the two evolutionary scenarios have a rather similar dependence of luminosities on the chemical composition, thus with the quoted systematic difference at any given metallicity.

"Sic stantibus rebus", the already quoted evidence that G98 evolutionary scenario appears able to nicely fit the Hipparcos mean magnitude of clumping He burning stars, implies that C99 must predict too luminous giants, running against the Hipparcos evidence. This has been confirmed by independent simulation of the clump population based on C99 evolutionary tracks, as transferred into the CM diagram by adopting model atmospheres by Castelli et al.(1997a,b). Data in Fig.1 gives the additional evidence that reasonable variations in the assumed original He content cannot decrease the C99 predicted luminosity by the required amount. Therefore one concludes that G98 evolutionary scenario works better.

However, for the sake of the discussion one has to notice that there is -at least in principle - a way to reconcile C99 prediction with Hipparcos observations. Fig. 4 in this paper shows that a substantial amount of mass loss could lower the C99 predictions by the required amount of about ΔlogL$\sim$0.1. As an example, one would require a mass loss by about 0.9 $M_{\odot}$ for a 2.0 $M_{\odot}$, and by about 0.6 $M_{\odot}$ for a 1.5 $M_{\odot}$ model. This, however, appear a too large requirement vis-a-vis current estimates for mass loss. Taking also into account the uncertainties on evolutionary parameters of the field population in the solar neighborhood, we regard the previous discussion not as a proof, but at least as a suggestion that the most updated models, as C99 are, when dealing with the progeny of degenerated RG tend to give too luminous He burning models.

5. Model differences

It appears of obvious relevance to address the problem of the discrepancies between the models by G98 and C99, trying to understand them in terms of different descriptions of the input physics. We leave out differences in the actual treatment of the input physics (e.g. interpolation in and between opacity tables), which we think are of smaller influence and might contribute to a minor part of the variations in the clump luminosities as shown in Fig. 2.

Since the most important quantity determining the core helium burning luminosity is the core mass at the helium flash, we will concentrate on this parameter. Fig. 5 shows that the 1 $M_{\odot}$ G98 models have core masses lower by $\approx 0.03\,M_{\odot}$ compared to C99 at the time of helium ignition at the RGB tip. Fig. 5 is made for solar composition but we checked that even for other metallicities and helium abundances the differences in the He core mass are very similar.

We have identified the following differences in the input physics used in the two evolutionary programs ("Padua" for G98 and "FRANEC" for C99 models) under consideration (other aspects such as reaction rates, electron screening, mixing-length formalism, etc., are to great extent identical):

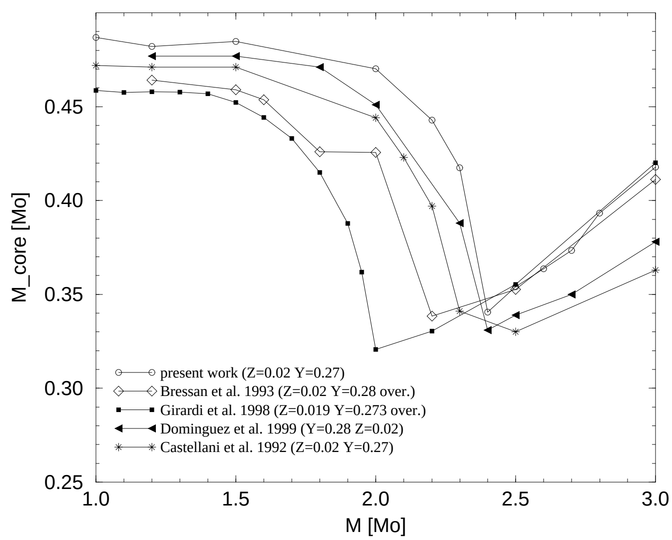


Fig. 5. Comparison between the helium core mass at helium ignition from G98 and C99 models. Chemical composition as labeled.

1. *Plasma neutrino emission*: G98 use Munakata et al. (1985) for $\log T > 7.7$, while C99 use Haft et al. (1994);
2. *Electron conduction*: G98 employ Hubbard & Lampe (1969), C99 Itoh et al. (1983);
3. *Radiative opacities*: the '92 tables by OPAL (Rogers & Iglesias 1992) are used in G98, while the '96 ones (Iglesias & Rogers 1996) are those in C99;
4. *Equation of state*: C99 use the new OPAL-EOS tables (Rogers et al. 1996) extended by that of Straniero (1988) in those regions where the OPAL-EOS does not exist (see Cassisi et al. 1998 for more details). G98 use their own analytic EOS, which takes into account partial ionization of hydrogen and helium, degeneracy and Coulomb-effects, and which has been described in Girardi et al. (1996).

The influence of some of these differences could be investigated quite easily because the two codes to some extent allow the selection of several sources of input physics. The tests were done for different cases of initial chemical composition and mass.

Neutrino emission. For the composition $Y = 0.27$, $Z = 0.02$ and an initial mass of $1.2\,M_{\odot}$ we find that M_c decreases from 0.482 to 0.476 (-0.006) $M_{\odot}$, if we switch from the Haft et al. (1994) back to the older Munakata et al. (1985) neutrino emission rates (FRANEC code). This is the same result as Cassisi et al. (1998; models 8 and

7 in their Table 1) obtained for an $0.8\,M_\odot$ model with $Y = 0.23$, $Z = 0.0001$ and also identical to what we find in the case of the Padua-code for the same model. In the latter case, we also verified that using the Munakata et al. (1985) emission down to $\log T = 7.4$ increases the He core mass by $\approx 0.001\,M_\odot$. Therefore, the total budget of core mass reduction due to the G98 treatment of neutrino emission physics amounts to $0.007\,M_\odot$. Recall that for the $1.2\,M_\odot$ model, the G98 value for M_c is $0.024\,M_\odot$ smaller than that of C99.

Opacities. We checked the effect of electron conduction opacity by switching in the FRANEC code from Hubbard & Lampe (1969) to Itoh et al. (1983) electron conduction opacities. The radiative opacities in these test cases are of a generation older than OPAL, but the differential effect of changing condution opacities can safely be assumed to be largely independent of the radiative opacities. The test model was a $1.5\,M_\odot$ star of $Y = 0.27$, $Z = 0.02$. The use of the older conductive opacities (i.e. those used by G98) leads to a reduction of $0.008\,M_\odot$ in M_c.

The influence of switching from the '92 to the '96 OPAL opacities was not tested, but according to our experience it should be minor compared to that of the electron conduction.

Equation of state. The EOS being a complicated part of both programs, we could not easily exchange one for the other. However, we could perform the following test: we took the pressure and temperature stratification of a C99-model on the RGB and applied the G99-EOS to it in order to obtain the density. We compared with the original C99 density stratification. The result is shown in Fig. 6 for a RGB model with Y=0.238, Z=0.004: while the core would have densities higher by about 1.5% in the Padua code, the envelope would be less dense by 1–2%. Both effects reflect the evolutionary changes along the RGB, such that a G98-model would appear to be more evolved than a C99 one. Therefore, also the difference in the EOS is expected to lead to a lower core mass at helium ignition for the G98 calculations.

From these three investigations we conclude that with the differences in neutrino emission rates and conductive opacities we can explain almost 60% of the discrepancy in core mass. This moves the G98 core masses already within the general spread of results. At least part of the remaining difference can be ascribed to the EOS.

Finally, we have compared results with three further codes, which use almost identical input physics as C99, in particular with regard to radiative opacities, neutrino emission and electron conduction. With regard to the Garching stellar evolution code (see, e.g., Schlattl & Weiss 1999) we find that M_c is $0.004\,M_\odot$ larger in the C99 models for a $0.8\,M_\odot$ star of $Y = 0.23$ and two metallicities, $Z = 0.0001$, 0.001. This small difference can be understood as being a consequence of the fact that more (0.004) helium is dredged up in the C99 models.

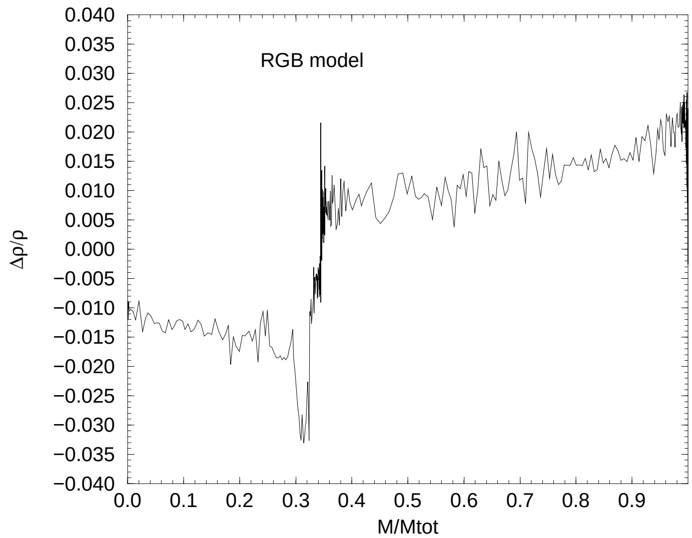
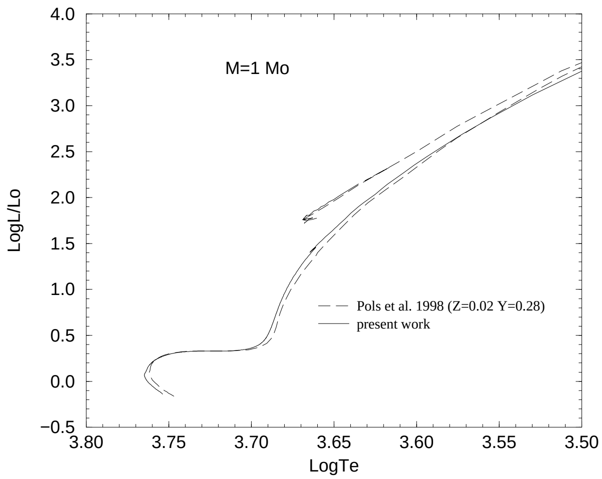


Fig. 6. Relative difference in density stratification between C99 and G98 EOS for an RGB model (see text).

Finally, evolutionary calculations by Pols et al. (1998) and Dominguez et al. (1999) with their respective codes (Pols et al. 1998; Straniero et al. 1997) reveal for various chemical compositions a very high degree of agreement with those of C99 as shown in Fig 7, including M_c, which differs by less than 0.01 $M_\odot$ between Dominguez and C99.

We therefore can conclude that more than half of the difference in the core mass at helium ignition between G98 and C99 can be removed by adopting the same neutrino emission rates and electron conduction opacities; using identical EOS would lead to further convergence, although we cannot quantify this point. At the same time, codes with identical physics do indeed result in very similar core masses which differ at a level of a few $10^{-3}\,M_\odot$ only. Therefore, the differences between the G98 and C99 results concerning this quantity are well understood as a consequence of different physical inputs. Note that taking from the literature (see e.g. Sweigart & Gross 1978) $\Delta \mathrm{logL/L_\odot} \approx 3.4\ \Delta M_c$ for stars with degenerate progenitors, **the difference in M_c explains the difference in luminosity among the various models in Fig.2, with the exception of predictions by Bressan et al. (1993) that reveal the contribution of some other difference than the core mass only.** As a conclusion, the different predicted luminosities we are dealing with appear as the natural results of evolutionary codes with different – but in both cases reasonable – input physics and thus as an example of the intrinsic unavoidable uncertainties in any current evolutionary scenario.



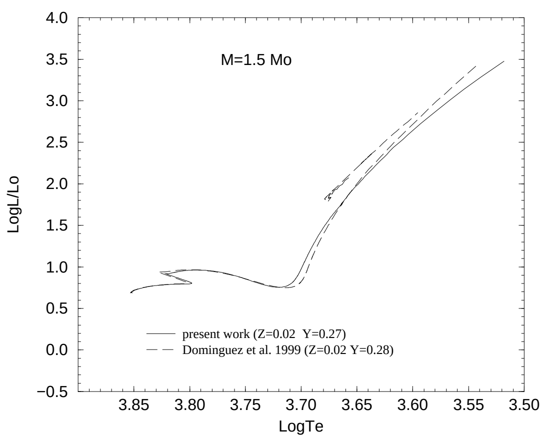


Fig. 7. Upper panel: comparison between the 1 $M_\odot$ model with solar compositions from Pols et al. (1998) and from C99; lower panel: comparison between the 1.5 $M_\odot$ model with solar compositions from Dominguez et al. 1999 and from C99.

6. Conclusions

As it is well known, and as recently explicitly reiterated, e.g., in Castellani (1999), the widespread use of the Henyey algorithm in producing stellar models assures that different codes with identical input physics must produce quite similar results. In this paper we presented several evidences, as taken from the literature, supporting such an occurrence. Thus evolutionary models are as "good" as the adopted input physics is. In this paper we have however shown that models based on different, but all "reasonable", physics inputs may have sizable differences in the predicted luminosity of He burning low mass stars.

Testing these theoretical predictions on the absolute magnitudes given by the Hipparcos satellite for He burning stars clumping in the field, one finds several indications for preferring the luminosity predicted by the Padua code, as used in Girardi et al. (1998). Even if the argument is still open to further investigation, we feel that the discussion presented in this paper raises - at least - serious doubts on the capability of the most recent "updated input physics" of correctly predicting the luminosity of He burning stars as originated from progenitors which underwent strong electron degeneracy. Therefore giving a serious warning about the uncritical acceptance of theoretical predictions concerning the luminosity of HB stars in galactic globulars and, in turn, all the related quantities.

In this respect, we note that observational evidences concerning the luminosity of globular HB stars are far from being clear. Caputo et al. (1999) have recently discussed RR Lyrae variables in the globular M5 showing that their pulsational properties possibly suggest a luminosity lower than predicted by C99 models. However, Cassisi et al. (1999) and Salaris & Weiss (1998) used HB stars brighter than G98 predictions to derive cluster distance moduli which appear in excellent agreement with some independent evaluations based on Hipparcos parallaxes for field subdwarfs. Here we can only conclude that the matter deserves further investigation, to decide whether or not the mass loss can reconcile "most updated" stellar models with Hipparcos constraints for the clump of He burning stars in the solar neighborhood.

Before closing the paper, let us notice that all the computations discussed in the paper agree in predicting He burning stars systematically brighter when the metallicity is decreased. The theoretical predictions are illustrated, e.g., in the figure 1 of G98, and figure 12 of C99. This has important implications for the method of distance determination based on clump stars, since it indicates that the clump population observed in nearby galaxies may have an intrinsic luminosity different from the local stars sampled by Hipparcos. In the case of the Magellanic Clouds, this difference may amount to 0.2 - 0.3 mag, as inferred by Cole (1998) and G98.

7. Acknowledgments

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